Enrollment No: _____________________ **Exam Seat No:**_____________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name :Mathematical Physics and Classical Mechanics

Subject Code :4SC05MPC1 **Branch: B.Sc. (Physics)**

Semester : 5 **Date :28/11/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Give basic mathematical formulae of the Beta(β) function by definition. 01

b) Give basic mathematical formulae of the Gamma(γ)function by definition. 01

c) Define: Fourier series giving general formula and Fourier coefficients. 01

d) Write any two applications of the Fourier series. 01

e) For which type of Fourier series, the coefficients $a_0 \& a_n = 0$ and $b_n \neq 0$? 01

f) Draw graph for the Fourier series $f(x) = \begin{cases} -3; 0 < x < 1 \\ 3; 1 < x < 2 \end{cases}$. 01

g) Write the value / formula of a_0 , a_n and b_n in the interval of $(-\ell,\ell)$ for the extended intervals. 01

h) Write any two properties of constraint forces. 01

i) What is virtual displacement? 01

j) What is virtual work? Write the formula for the Virtual Work Principle. 01

k) What are the ignorable and cyclic coordinates? 01

l) Define variational principle. 01

m) List any three advantages of Lagrangian formulation 01

n) Define the linear differential equation. 01

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(A) Prove the following for the β function, where $p > 0$ and $q > 0$; **09**

(1) $\beta(p,q) = \int_0^{\infty} \frac{y^{p-1}}{(1+y)^{p+q}}\, dy.$

(2) $\beta(p,q) = 2\int_0^{\pi/2} sin^{2p-1}\theta . cos^{2q-1}\theta . d\theta.$

(3) $\beta(p,q) = \int_0^{1} \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\, dx.$

(B) Obtain the Fourier series of $f(x) = \begin{cases} 0 \ ; \ 0 < x < l \\ a \ ; \ \ell < x < 2\ell \end{cases}$; Draw graph. **05**

Q-3 **Attempt all questions** (14)

(A) Prove the following for the γ function.Where,$p > 0$, $a = Constant, x = y$. 12

(1) $\gamma(p) = a^p \int_0^\infty e^{-ax}\, x^{p-1}\, dx$

(2) $\gamma(p) = 2\int_0^\infty e^{-x^2}\, x^{2p-1}\, dx.$

(3) $\gamma(p) = \frac{1}{p}\int_0^\infty e^{-x^{1/p}}\, dx.$

(4) $\gamma(p) = \int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{p-1} dy$

(B) Write Dirichlet conditions for periodic function in Fourier series expansion. 02

Q-4 **Attempt all questions** (14)

(A) Find the Fourier series for the periodic function e^{-x}in the interval $(-L, L)$. 06

(B) From the Fourier expansion of $f(x) = x^2$in the interval $(-\pi, \pi)$, prove that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. 08

Q-5 **Attempt all questions.** (14)

(A) What are the constraints. Give its types with examples. 07

(B) Write a short note on: Generalised coordinates. 07

Q-6 **Attempt all questions** (14)

(A) Derive step-by-step: Lagrange's Equation of Motion for the Non-Conservative System. 10

(B) Derive equation for the Simple Harmonic Motion from the Lagrangian Function / Formulation. 04

Q-7 **Attempt all questions** (14)

(A) Write the final formula equation for the motion of simple pendulum, Compound pendulum and spherical pendulum and double pendulum. Derive any one of these formulae using Lagrangian formulation. 08

(B) What is Hamilton's principle? Derive: Lagrange's Equation from Hamilton's Principle 06

Q-8 **Attempt all questions** (14)

(A) Establish equivalence of Newton's equation and Lagrange's equation. 04

(B) Derive Hamilton's principle from Newton's equation. 06

(C) Describe in detail: Lagrange's undetermined multipliers. 04